[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 5805 H

Unique Paper Code : 237352

Name of the Paper : Real Analysis

Name of the Course : **B.Sc. (Hons.) Statistics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Q. No. **1** is compulsory.
3. Attempt **six** questions in all.

1. (a) Write the Supremum and Infimum of the set

$$S = \left\{1 - \frac{1}{n}, n \in N\right\}$$

(b) (i) Give an example of a set which is a neighbourhood of each of its points.

(ii) Give an example of a set which is neither an interv[a]
nor an open set.

(c) Show that arbitrary union of closed sets need not b[e]
closed.

(d) Examine the convergence of the series $\sum \cos\frac{1}{n}$.

(e) What is an alternating series? Give an example.

(f) Examine the continuity of the function $f(x) = [x],\ x \in$ [
at $x = 2$.

(g) Find the value of c of Lagrange Mean Value Theore[m]
for the function

$$f(x) = 1/x \quad \text{on } [1\ \ 4]$$

(2,2,2,2,2,3,2

2. (a) Define a lower bound and an infimum of a non-empt[y]
bounded set S of real numbers. Prove that a real numb[er]
t is the infimum of S iff

(i) $x \geq t \quad \forall\ x \in 5$

(ii) for each $\in > 0$, there is a real number $x \in$
such that $x < t + \in$.

(b) Define neighbourhood of a point and open set. Prov[e]
that the intersection of two open sets is open. (6,6

3. (a) State and prove Monotone Convergence Theorem.

(b) Prove that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ exists and lies between 2 and 3.

(6,6)

4. (a) State Cauchy's first theorem on limits. Use it to show that

$$\lim_{n\to\infty}\frac{1+2^{1/2}+3^{1/3}+\ldots+n^{1/n}}{n}=1$$

(b) Define

(i) a convergent series,

(ii) a divergent series.

Show that the series $1 + r + r^2 + \ldots + \ldots$ converges if $0 < r < 1$.

(6,6)

5. (a) Let $\sum u_n$ be a positive term series such that $\lim_{n\to\infty}\frac{u_n}{u_{n+1}} = l$. Prove that the series $\sum u_n$ converges if $l > 1$. What happens if $l = 1$?

P.T.O.

(b) Examine the convergence of the following series.

(i) $\sum_{n=1}^{\infty}\left(n^{1/n}-1\right)^{n}$

(ii) $\frac{1}{5}+\frac{\sqrt{2}}{7}+\frac{\sqrt{3}}{9}+\frac{\sqrt{4}}{11}+\ldots$ (6,6)

6. (a) Define conditionally convergent series. Show that the series

$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots$ is conditionally convergent.

(b) Show that the function f defined by

$f(x) = |x| + |x-1|$ is continuous but not derivable at $x = 0$, $x = 1$. (6,6)

7. (a) State and prove Rolle's Theorem.

(b) Obtain Maclaurin's series expansion of cos x.

(c) Find the value of a that will make

$\lim_{x\to 0}\left(\frac{\sin 3x - a\sin x}{x^3}\right)$ finite and hence find the limit.

(5,4,3)